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A NOTE ON THE INTERPRETATION OF THE INTERCEPT OF COBB-DOUGLAS TYPE OF PRODUCTIONS THROUGH DIMENSIONAL ANALYSIS

by

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Sinopsis

Kertas ini mengemukakan bahawa interpretasi terhadap faktor silangan (intercept) dalam fungsi pengeluaran Cobb-Douglas sebagai pekali teknologi sebenarnya mempunyai sebab-musabab yang tertentu. Analisis dimensi telah menerangkan bahawa jika fungsi pengeluaran Cobb-Douglas itu mempunyai dimensi yang homojenis maka setentunya pemalar A mempunyai sesuatu dimensi yang tertentu yang jelas akan bergatung kepada dimensi-dimensi faktor y, L dan k. Inilah yang disifatkan oleh kertas ini sebagai "variable content" kepada faktor A.

Synopsis

This paper shows that the interpretation of the intercept of the Cobb-Douglas production function referred to as a techonological coefficient has its reasons and backing. Dimensional analysis has shown that if a Cobb-Douglas production function is dimensionally homogenous, then the intercept or constant A must neccessarily have a certain dimension which of course depends on the dimensions of factors y, l dan k. This is what the paper referred to as the "variable content" of factor A.

Introduction

Majority of economic researchers employ multiple regression analysis or some other econometric techniques to evaluate and estimate certain parameters in their models. In fact, many 'modern' researchers are inclined to suggest that without certain mathematical tools, the analyses, more problems are introduced, especially the problems of identification and estimation. For instance, the seemingly uncorrelated variables like number of road accidents and total number of students in a university can prove to have some interrelatedness if we were to apply regression analysis. This has raised doubts as to whether conclusions derived by the use of regression analysis acceptable or adequate.

While this short note will not question or discredit this usage, it is to our advantage to understand and foresee the effects of certain mathematical orientation towards specification and then estimation. This is usually so in economic researches where there is no clear demarcation between endogenous and exogeneous variables. In one equation a variable is endogeneous and in another it is exogeneous. This simultaniety problem is not uncommon. It has also been acknowledged that economic data are usually highly intercorrelated with one another. Thus on applying multiple regression techniques, we will normally encounter problems of multicollinearity and to some extent autocorrelation of the random error term and other related econometric problems. The effects of the introduction of these problems is universally known.

Though it has already been shown that problems of multicollinearity in regression analysis can be overcome by employing suitable techniques, for example, by principal components analysis, stepwise regression analysis and the like, there is no guarantee that variables in the equation are correctly specified. This is especially so if we were to 'tackle' the problem in terms of its dimensionality. For example, in group or matrix theory (which we usually use without realising it) two matrices having different dimensions are non-additive and sometimes non-multiplicative. The fact that addition and multiplication of variables commonly done in our researches without due consideration of their dimensionality are often overlooked. Thus, it seems that our application of regression analysis to these data has some time or other deviated from the preconditions set the theory of matrices. How far this is true is subject to further scrutiny. But one thing for certain is that the use of principal components analysis or stepwise regression analysis does not guarantee the homogeniety of the dimensions of variables under consideration.

It is therefore possible that this could be the reason why we often overlook the necessary meaning of one of the most often estimated but 'neglected' parameters in some or many of our studies. It is none other than the intercept or the regression constant. It is possible that the same data analysed by two persons will give *dissimilar* figure on this parameter though other parameters are similar. While in the studies of production function, a simple one is the being Cobb-Douglas type, this parameter is interpreted as technological parameter, it is sometimes ponderable why technology changes as the researcher changes, while time factor remains unaltered. This indicates that an error must have been committed, either by the different researchers or misspecification of variables or the equation that is estimated is not dimensionally homogenuous. We will only consider the last possibility, and in any case, it is also subject to error. Given that this parameter is a technological index, there is no evidence of satisfactory explanation as to why it is so. This note will try to demonstrate the "variable content" of this parameter so that misinterpretation can be minimised.

The Concept of Dimensional Analysis

De Jong (1976) postulates that dimensions are concerned with the units in which quantities are measured. For example, an equation is dimensionally homogenous if the measurement unit on the left side of an equation equal to the units on the right side. In consederig this, let us look at the formula for area which is given by

$$\text{Area} = \text{length} \times \text{breadth} \qquad (1)$$

Equation (1) is dimensionally homogeneous because the unit measurement of area is in square while unit measurement of length multiply breadth is also in square, irrespective of whether in square inches, metres, etc.

It is foreseen that we will face certain problems in applying this concept in economics. However, this will not hinder the need to "correctly" specify economic equation in terms of their dimensionality. Firstly, let us look at examples where dimensional analysis originates. As we have already perceived, dimensional analysis is a branch of mathematics which is mainly concerned with the analysis of dimensions of quantities. Classical mechanics, physics and chemistry usually employ this technique to analyse certain phenomena. The most conspicuous one is the classical mechanics. In this discipline, all quantities are claimed to be expressible in terms of three *primary* or *fundamental* dimensions, namely mass [M], length [L] and time [T]. Thus, for example, if we were to measure velocity which is distance divided by time, then the unit measurement is $\frac{L}{T}$ or LT^{-1}. This new dimension will be called *secondary* or *derived* dimension.

Turning towards economic theory, it makes us wonder as to whether there exists such a dimension. In fact, there may be factors having no dimension at all. As such, it makes sense to question whether this branch of mathematics is applicable to the study of economic problems.

The Concept of Primary and Secondary Dimensions in Economics*

In the choice of dimensions especially a primary dimension we will

*This section only relays the ideas set by De Jong (1976).

be faced by questions like is it true, how and why. Since economics is far from mechanics it is not foreseeable that we can relate the concept of mass, length and time to satisfy our purpose. It is almost impossible for economists to define just one set of primary dimensions for use in every case. For example, primary dimensions for microeconomic analysis differ from primary dimensions for macroeconomics. One probable primary dimension that is common in micro and macro is the time factor [T].

Let us now consider Irving Fisher's equation of exchange, MV = PQ where M denotes net stock of money, V velocity of circulation, P general price level and Q flow of goods. In considering the dimensionality of this equation, let us first of all look at the factor Q, i.e. the flow of goods. Since there is no distinction made between the types of goods like clothing, cars, rice, labour, potatoes etc, and this equation considers them to have additive quantities, we only have one dimension of goods. Let us denote its dimension by [R]. The next plausible assumption is that any given nation has only one monetary system. Then we have one dimension for money, [M]. The product MV represents the sum total of all payments per unit time, so that we also need the dimension for time [T]. In total, we have three primary dimensions [M], [R] and [T] in considering Fisher's equation of exchange.

Following the simplest Keynesian Model, goods can be devided into two distinct categories i.e. consumption goods and investment goods and of course we have another factor, labour. Consequently, it is no longer possible for all the goods to have one real dimension. We now have to define dimensions for each of different type of goods say consumption good $[R_e]$, investment or capital goods $[R_k]$ and labour $[R_a]$. Thus, we now have five primary dimensions. Categorising further, we can define more primary dimensions, for example along the lines of Cassel and Pareto.

Going back to the formula MV = PQ we have the following,

$$
\begin{aligned}
M &\in [M] \\
V &\in [T^{-1}] \\
P &\in [MR^{-1}] \\
Q &\in [RT^{-1}]
\end{aligned}
$$

The last two factors that have secondary dimensions is not due to any incident. As we are all aware, general price level is a match between supply of money and supply of goods and so also the flow of goods is a match between availability of goods over time. On discussing Fisher's equation of exchange's dimensionality, we arrive at matching their dimensions, i.e.

$$\begin{aligned} MV &= PQ \\ \Rightarrow [M][T^{-1}] &= [MR^{-1}][RT^{-1}] \\ \Rightarrow [MT^{-1}] &= [MR^{-1}\ RT^{-1}] \\ &= [MT^{-1}] \end{aligned}$$

which indicates that the equation is dimensionally homogenous. The question that we may now ask is that: what do we want to achieve by this? As we have seen, nothing spectacular has been shown except that the verification of Fisher's equation of exchange is dimensionally homogeneous.

On Interpretation of the Intercept of Cobb-Douglas Type Production Function

Let us first consider a simple money flow formula represented by

$$y = \alpha x$$

where y represents money flow
x represents stock of money
and α represents parameter (abstract)

By inspection, this formulation seems erroneous because a flow cannot be equal to stock. Our intuition tells us that when we talk about flow, we always associate it with the time factor. Since no time factor is included in this formulation, it will certainly become illegitimate. It is of course equatly ilegitimate to reverse the statement i.e. $x = \beta y$. [It is notable that some equations in economics are reversible especially where simultaneours equations problems occur]. It must be cautioned however that there is no reason to condemn a certain equation as erroneous because in economics, only economics can determine the right economic equations, in physics, only physics can determine them ad so forth. Dimesional analysis is no substitute for economic theory but it only helps to locate possible "specification error". Ultimately, it is the economic theory that decides whether an equation is correct or erroneous.

Dimensional analysis have shown that the above equation is wrongly specified and the possible source of misspecification is that the time factor is not included. It is not always true that economics neglected the time factor. This is manifested in many dynamic equations like income, labour, etc. Now, let us dynamicised the above eqation to become,

$$y_t = \alpha x_t \qquad (2)$$

where t denotes the time factor.

Our problem of nonhomogeneity of dimension is still not solved, because

$$Y_t \in \frac{[M]}{[T][T]} = [MT^{-2}]$$

$$x_t \in \frac{[M]}{[T]} = [MT^{-1}]$$

$\alpha \in [1]$ where [1] denotes dimensionless identity.

One possible way to solve this nonhomogeniety problem is to define factor α having dimension $[T^{-1}]$. This is logical due to the fact that α changes over time. If this is true, equation (2) is now dimensionally homogenous. It is economics that determines the truthfulness of this contention.

Let us now consider a simple Cobb-Douglas production function having two inputs, i.e. labour and capital. That is,

$$y = AL^{\alpha}K^{1-\alpha} \qquad (3)$$

where y = output

L = labour (measured in man-hours per unit time)

K = stock of real capital

α and A = parameters to be determined.

This dimensional consistency between the LHS and RHS of equation (3) is achieved through the constant A, the units of which will depend upon the units used for measuring factors L and K.

This is a rather rough type of production where no distinction is made between different kinds of final products included in y. Thus we only have one dimension for y, denote it by $[R_e]$, one dimension for labour, $[R_e]$ and one dimension for stock of real capital $[R_k]$. We can then write.*

$$y \in [R_eT^{-1}]$$
$$L \in [R_aT^{-1}]$$
$$K \in [R_k]$$

*This is due to the recognition that y and L changes at faster rate than stock of capital over time. We can also say that $K\,\alpha[R_kT^{-1}]$, but the conclusion will be similar.

In order to form a 'meaningful' equation, the factor A must neccessarily have a certain form of dimension. The dimension of A can be computed from

$$A = \frac{y}{L^{\alpha} K^{1-\alpha}}$$

so that

$$A \in \frac{[R_e T^{-1}]}{[R_a T^{-1}]^{\alpha} \ [R_k]^{1-\alpha}} = [R_e R_a^{-\alpha} R_k^{\alpha-1} T^{\alpha-1}]$$

This suggests that a dimensionally homogeneous Cobb-Douglas production function stipulates that the parameter A is governed by output, labour, capital and time influences. Consequently, if the variables y, L and K are experessed as index numbers, which is dimensionless, then A is also dimensionless.

Conclusion

As we have already known or accepted, the coefficient or parameter A of the Cobb-Douglas production function represents technical knowledge or the "state of arts". This short paper, with the help of dimensional analysis is able to point out the "variable content" of this parameter. While the result is nothing new, in the sense that parameter A is of course of technological parameter, it makes us satisfied that there is sufficient explanation as to why it is so, deductively. Dimensional analysis tends to suggest that economic equations should be a dynamic one. It is up to the economists to accept or nullify this suggestion.

Reference

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